Nonlinear Functional Analysis
Tuesday-Thursday, Spring 2020

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Background Material

We will use the Lebesgue integral, some basic results of linear functional analysis about linear pde's, and some results from topology. Precise descriptions and motivations for these results, together with references, will be provided. Useful references are *Real Analysis* by Royden and Fitzpatrick, *Linear and Nonlinear Functional Analysis with Applications* by Phillippe Ciarlet, and *Functional Analysis, Sobolev Spaces, and Partial Differential Equations* by Haim Brezis A good idea of the flavor of the course can be found on my review of Ciarlet's book: SIAM Review, Vol 58 (2016).

1 Differentiation of Nonlinear Operators

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1. The Fréchet Derivative: Inverse Function Theorem, Implicit Function Theorem.
2. Differentiability for operators on Sobolev and Schauder spaces
3. Lyapunov-Schmidt Reduction and the Crandall-Rabinowitz Theorem.
4. A few applications to PDE's

2 The Brouwer Degree and Applications

1. Motivation by the winding number and Rouché's Theorem
2. Four properties that uniquely define the Brouwer Degree
3. Geometric Applications of the Degree: Brouwer Fixed-Point theorem, the Invariance of Domain, Borsuk's Theorem, Hairy Ball Theorem
4. Analytic construction of the degree using Sard's Theorem and a lemma of Milnor

3 Degree for mappings on Infinite Dimensional spaces

1. Two Basic Obstacles: a lemma of Riesz and a theorem of Kuiper
2. Construction and properties of the Leray-Schauder degree
3. Galerkin approximations and strongly monotone mappings
4. Applications of degree to existence of solutions of pde's
5. Bifurcation Problems: the Rabinowitz Global Bifurcation Theorem
6. Linear and nonlinear Fredholm operators; The oriented degree for proper nonlinear Fredholm operators with some applications.